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# Pulsed Laser Backscatter

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November 1980

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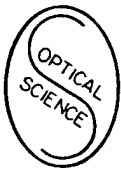
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## ABSTRACT

In certain remote sensing applications, it is of interest to determine the power backscattered from air molecules when a pulsed laser beam propagates through the atmosphere. Our primary concern here is the interaction between the laser pulse and the atmosphere at altitudes above the region where turbulence effects are appreciable. The analysis indicates that a significant fraction of the laser pulse is backscattered into a  $1 \text{ m}^2$  detector if the pulse is observed as it propagates from an altitude of 20 km to an altitude of 30 km. Analysis and numerical results are presented.

The analysis begins with the equation that describes the reduction in the energy associated with the laser pulse as it propagates through the scattering medium

$$E(z_2) = E(z_1) \exp \left[ - \int_{z_1}^{z_2} dz \beta(z) \right] \quad . \quad (1)$$

Here  $E(z)$  is the energy in the pulse at altitude,  $z$ , and  $\beta(z)$  is the total scattering coefficient, which is assumed to be a function of altitude. The total energy scattered out of the laser beam,  $E_s$ , is then given by

$$E_s = E(z_1) \left\{ 1 - \exp \left[ - \int_{z_1}^{z_2} dz \beta(z) \right] \right\} \quad . \quad (2)$$

We are interested in the case when  $E(z_1) = 1$  joule. Therefore, only the quantity,  $\beta(z)$ , need be evaluated to determine the total energy scattered out of the beam.

We assume that the only scattering mechanism is Rayleigh scattering by air molecules. Thus, an evaluation of Rayleigh scattering will lead to a determination of  $\beta(z)$ . When it is assumed that the scattering is incoherent, the angular scattering coefficient,  $\beta(\phi)$ , is given by<sup>1</sup>

$$\beta(\phi) = \frac{\pi^2 (n^2 - 1)^2}{N \lambda^4} \sin^2 \phi \quad , \quad (3)$$

where  $\phi$  is the scattering angle and equals  $90^\circ$  in the forward direction, and  $270^\circ$  is the backward direction. In addition,  $n$  is the refractive index,  $N$  is the number of molecules per unit volume, and  $\lambda$  is the wavelength. The total scattering coefficient,  $\beta$ , is obtained by integrating  $\beta(\phi)$  over the solid angle

$$\beta = \int d\Omega \beta(\phi) \quad . \quad (4)$$

We recognize that  $\int d\Omega = 2\pi \int_0^\pi d\phi \sin \phi$  and Eq. (4) becomes

$$\beta = \frac{8\pi^3(n^2 - 1)^2}{N \lambda^4} \int_0^\pi d\phi \sin^3 \phi \quad . \quad (5)$$

The integral is a standard form and equals 4/3, therefore, the total scattering coefficient is given by the expression

$$\beta = \frac{8\pi^3(n^2 - 1)^2}{3N \lambda^4} \quad . \quad (6)$$

We will be interested in the scattering mechanism over a wide range of altitudes and for this reason it is necessary to determine how  $\beta$  varies with increasing altitude. The quantity  $n^2 - 1$  is proportional to  $N$ , the number of molecules per unit volume. Therefore,  $\beta$  is proportional to  $N$ . From the perfect gas law  $N$  is proportional to pressure and inversely proportional to temperature. Therefore, if  $\beta$  is known at one altitude, its value at other altitudes is given by

$$\beta(z) = \frac{8\pi^3(n^2 - 1)^2}{3N \lambda^4} \frac{P(z)}{P_0} \frac{T_0}{T(z)} \quad , \quad (7)$$

where  $P_0$  and  $T_0$  are the original pressure and temperature at which the total scattering coefficient is known, and  $P(z)$  and  $T(z)$  are the pressure and temperature at the desired altitude. The total energy scattered out of the beam is then

$$E_s = E(z_1) \left\{ 1 - \exp \left[ - \frac{8\pi^3(n^2 - 1)^2}{3N \lambda^4} \frac{T_0}{P_0} \int_{z_1}^{z_2} dz \frac{P(z)}{T(z)} \right] \right\} \quad . \quad (8)$$

When it is assumed that this energy is uniformly distributed over a sphere centered at an altitude of  $(z_1 + z_2)/2$  the energy incident on a receiver with area of  $1 \text{ m}^2$  is obtained by multiplying  $E_s$  by the solid angle subtended by the receiver. The energy received,  $E_R$ , is then

$$E_R = E_s \left( \frac{z_1 + z_2}{2} \right)^2 \quad (9)$$

We are mainly concerned with wavelengths in the visible regions of the spectrum where photon counting detectors such as photomultipliers are available. It is, therefore, more appropriate to determine the number of photons,  $N_p$ , collected by the receiver aperture. The energy per photon is equal to  $hc/\lambda$  where  $h$  is Planck's constant and  $c$  is the speed of light. Thus,

$$N_p = \frac{E_s \lambda}{hc \left( \frac{z_1 + z_2}{2} \right)^2} \quad (10)$$

Eq. (10) along with Eq. (8) are the primary results of the analysis, the total energy scattered out of the laser beam and the total number of photons collected by the receiver aperture can both be determined. The results for various wavelengths in the visible region of the spectrum, assuming a one joule incident pulse, are summarized in Fig.'s 1 and 2. To obtain these results the following values of the parameters were used

$$n = 1.000293 \quad , \quad (11a)$$

$$N = 2.547 \times 10^{25} \text{ m}^{-3} \quad , \quad (11b)$$

$$P_0 = 1013.25 \text{ mb} \quad , \quad (11c)$$

$$T_0 = 288.15^\circ \text{ K} \quad , \quad (11d)$$

$\lambda$ ( $\mu$ )	$z_1 = 10$ km $z_2 = 20$ km		$z_1 = 15$ km $z_2 = 25$ km		$z_1 = 20$ km $z_2 = 30$ km	
	Fractional Power Loss into $4\pi$ steradians	Number Photons into $1\text{m}^2$ aperture/ joule of Incident Beam	Fractional Power Loss into $4\pi$ steradians	Number Photons into $1\text{m}^2$ aperture/ joule of Incident Beam	Fractional Power Loss into $4\pi$ steradians	Number Photons into $1\text{m}^2$ aperture/ joule of Incident Beam
1	$1.9642 \times 10^{-3}$	$4.3890 \times 10^7$	$8.9402 \times 10^{-4}$	$1.1237 \times 10^7$	$4.0440 \times 10^{-4}$	$3.2531 \times 10^6$
.7	$8.1728 \times 10^{-3}$	$1.2783 \times 10^8$	$3.7219 \times 10^{-3}$	$3.2747 \times 10^7$	$1.6840 \times 10^{-3}$	$9.4826 \times 10^6$
.6	$1.5256 \times 10^{-2}$	$2.0454 \times 10^8$	$6.9190 \times 10^{-3}$	$5.2179 \times 10^7$	$3.1198 \times 10^{-3}$	$1.5058 \times 10^7$
.5	$3.1895 \times 10^{-2}$	$3.5635 \times 10^8$	$1.4298 \times 10^{-2}$	$8.9857 \times 10^7$	$6.4901 \times 10^{-3}$	$2.6104 \times 10^7$
.4	$7.9667 \times 10^{-2}$	$7.1206 \times 10^8$	$3.5523 \times 10^{-2}$	$1.7860 \times 10^8$	$1.5919 \times 10^{-2}$	$5.1223 \times 10^7$
.3	$3.7412 \times 10^{-1}$	$1.8376 \times 10^9$	$1.1032 \times 10^{-1}$	$4.1599 \times 10^8$	$5.1183 \times 10^{-2}$	$1.2352 \times 10^8$

Figure 1. Atmospheric Scattering Results for Three Different Altitude Ranges.

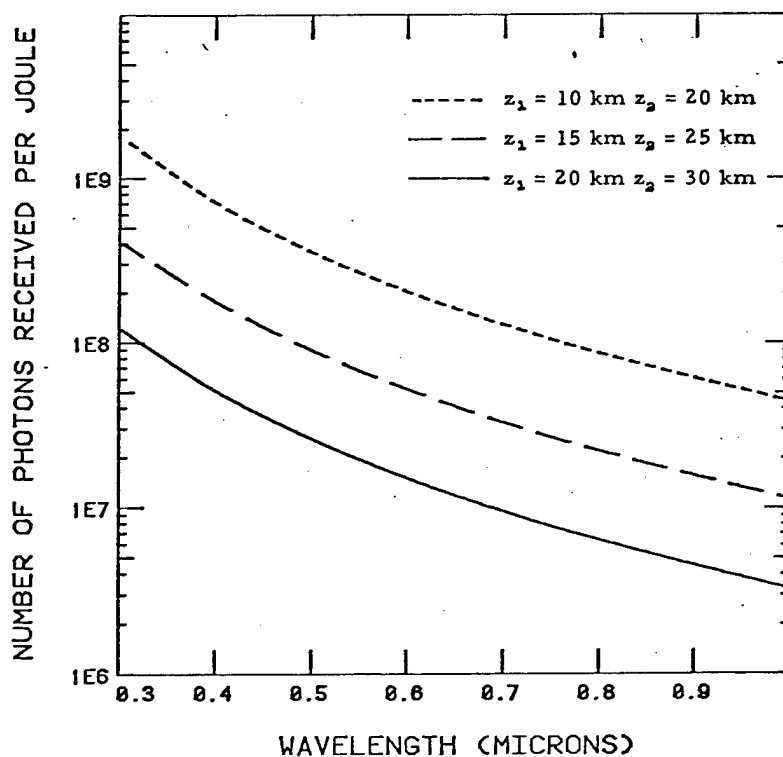


Figure 2. The Number of Photons Incident Upon a  $1\text{m}^2$  Receiver Area for a One Joule Incident Pulse.

These results ignore atmospheric attenuation between the receiver and altitude  $z_1$ . Vertical propagation is assumed.



where the values for the first two quantities are appropriate for air at sea level temperature and pressure conditions, and were obtained from Ref. 1. In addition to  $P_0$  and  $T_0$ , the values of  $P(z)$  and  $T(z)$  used to evaluate the integral in Eq. (8) were obtained from Ref. 2 .

Fig. 1 summarizes the results for the actual wavelengths that were used. Fig. 2 is a plot of Eq. (10) for each of the three altitude ranges considered, indicating that for a wavelength of  $1 \mu$  over three million photons are available if the atmospheric scattering is observed from 20 to 30 kilometers in altitude. This is a significant number and can easily be detected with existing devices.

## References

1. Earl J. McCartney, Optics of the Atmosphere, John Wiley and Sons (New York 1976), Chapter 4.
2. U.S. Standard Atmosphere Supplements, 1966, Prepared under the sponsorship of Environmental Science Services Administration, National Aeronautics and Space Administration, and the United States Air Force. Table 5.1, Mid-Latitudes, Spring/Fall, p 118.